Close Tue: 15.1, 15.2
Close Thur: 15.3

15.2 Double Integrals over General Region

Last time:

For the rectangular region, *R*:

$$a \leq x \leq b \ , \qquad c \leq y \leq d$$

we learned

$$\iint_R f(x,y)dA$$

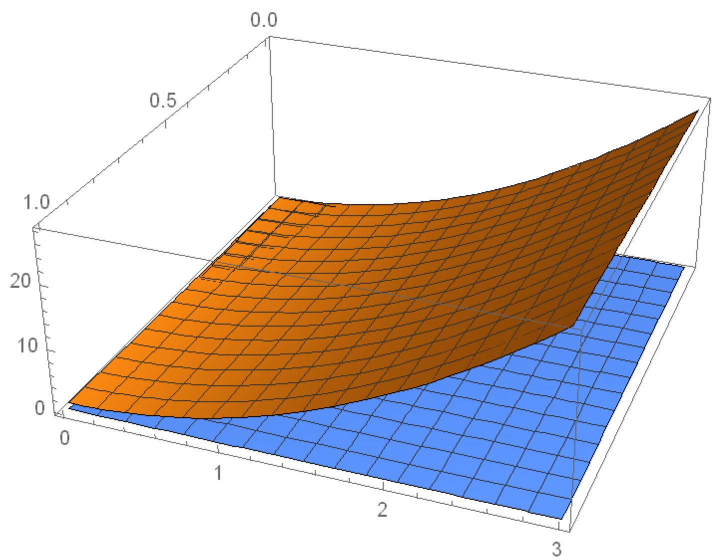
$$= \int_a^b \left(\int_c^d f(x,y)\, dy \right) dx$$

$$= \int_c^d \left(\int_a^b f(x,y)\, dx \right) dy$$

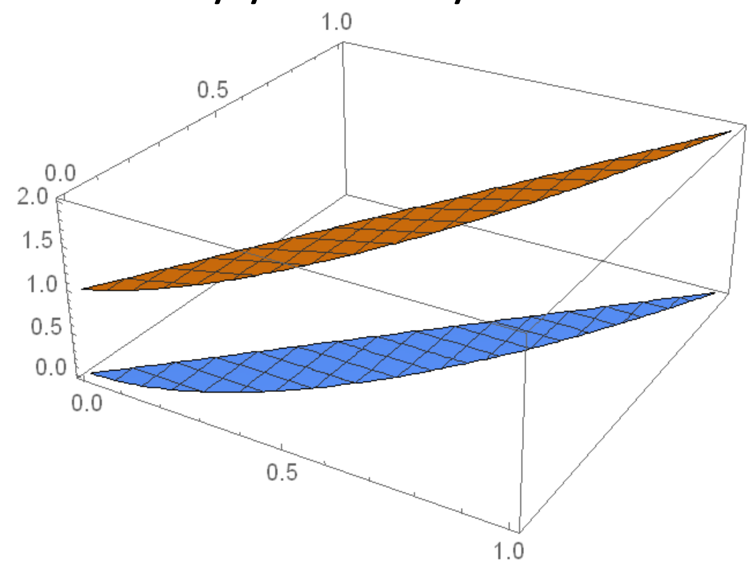
Type 1 (Top/Bot)	Type 2 (Left/Right)
For all *x* in the range, $a \leq x \leq b$, we have $g_1(x) \leq y \leq g_2(x)$	For all *y* in the range, $c \leq y \leq d$, we have $h_1(y) \leq x \leq h_2(y)$
$\int_a^b \left(\int_{g_1(x)}^{g_2(x)} f(x,y)\, dy \right) dx$	$\int_c^d \left(\int_{h_1(y)}^{h_2(y)} f(x,y)\, dx \right) dy$

In 15.2, we discuss regions other than rectangles.

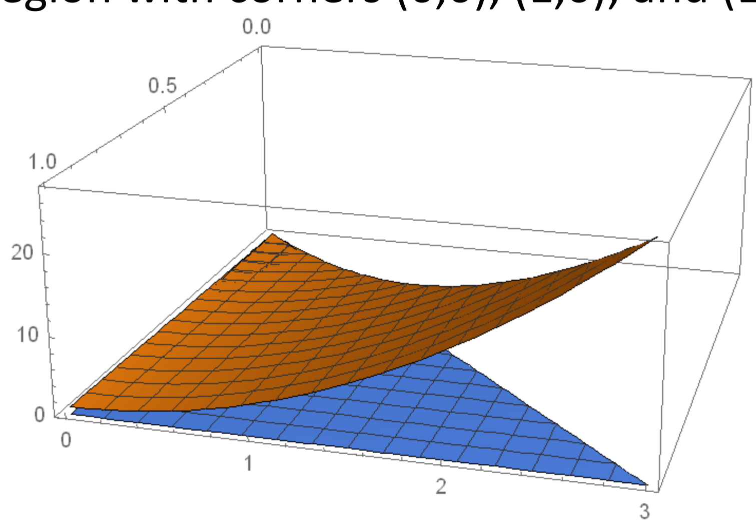
The surface $z = x + 3y^2$ over the rectangular region $R = [0,1] \times [0,3]$



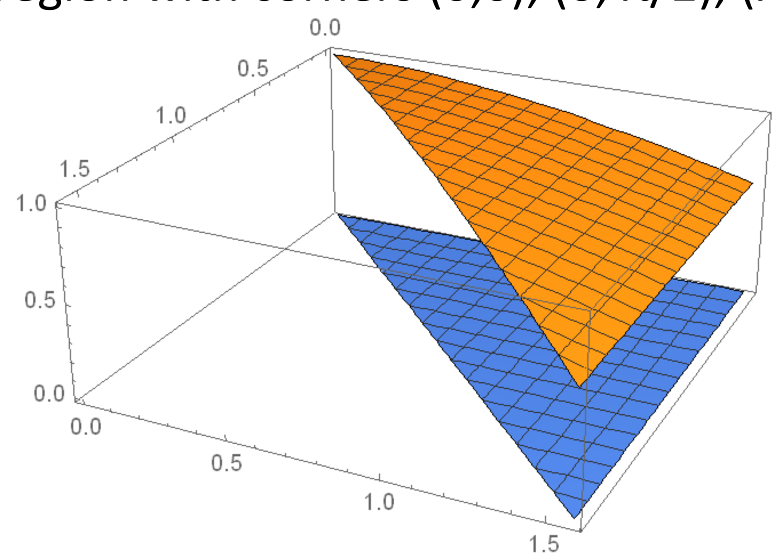
The surface $z = x + 1$ over the region bounded by $y = x$ and $y = x^2$.



The surface $z = x + 3y^2$ over the triangular region with corners (0,0), (1,0), and (1,3).



The surface $z = \sin(y)/y$ over the triangular region with corners (0,0), (0, π/2), (π/2, π/2).



Examples:

1. Let D be the triangular region in the xy-plane with corners (0,0), (1,0), (1,3).

   Evaluate $\iint_D x + 3y^2 \, dA$

2. Find the volume of the solid bounded by the surfaces $z = x + 1, y = x^2$, $y = 2x, z = 0$.

3. Draw the region of integration for

$$\int_{0}^{\pi/2}\int_{x}^{\pi/2} \frac{\sin(y)}{y}\, dy\, dx$$

then switch the order of integration.

4. Switch the order of integration for

$$\int_0^4 \int_{\sqrt{x}}^2 \sin(y^3)\, dy dx$$

Setting up a problem given in "words":

1. Find integrand

Solve for "z" anywhere you see it.

If there are two z's, then set up two double integrals (subtract at end).

2. Region?

Graph the region in the *xy*-plane.

a) Graph all given *x* and *y* constraints.

b) And find the xy-curves where the surfaces (the z's) intersect.

Examples (directly from HW):

HW 15.2: Find the volume enclosed by $z = 4x^2 + 4y^2$ and the planes $x = 0$, $y = 2$, $y = x$, and $z = 0$.

HW 15.3:

Find the volume below $z = 18 - 2x^2 - 2y^2$ and above the xy-plane.

HW 15.3:

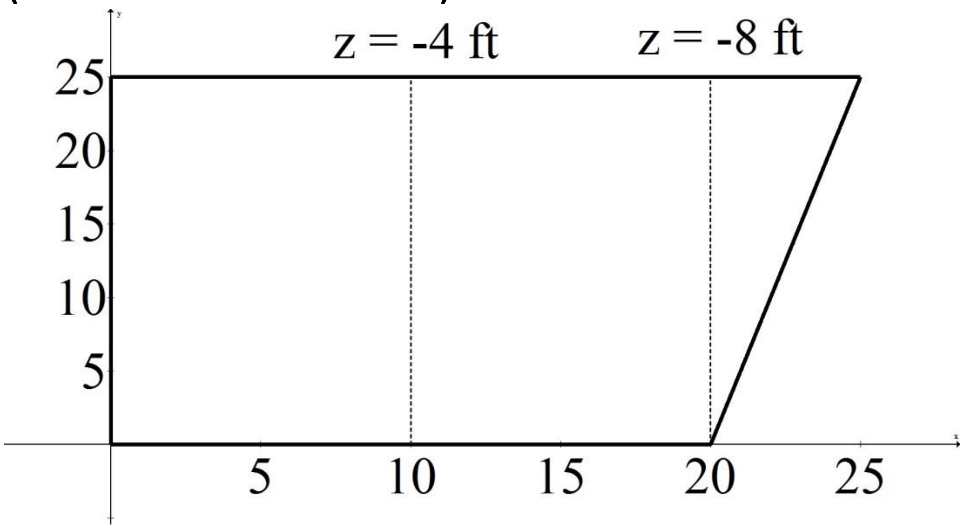
Find the volume enclosed by
$-x^2 - y^2 + z^2 = 22$ and $z = 5$.

HW 15.3:

Find the volume above the upper cone
$z = \sqrt{x^2 + y^2}$ and
below $x^2 + y^2 + z^2 = 81$

An applied problem:

Your swimming pool has the following shape (viewed from above)



The bottom of the pool is a plane with depths as indicated (the pool gets deeper in a linear way from left-to-right)

Solution:

1. **Surface?**

   Slope in y-direction = 0

   Slope in x-direction = -4/10 = -0.4

   Also the plane goes through (0, 0, 0)

   Thus, the plane that describes the bottom of the pool is: **z = -0.4x + 0y**

2. ***Region?***

   The line on the right goes through (20,0) and (25,25), so it has slope = 5 and it is given by the equation

   y = 5(x-20) = 5x – 100

   or **x = (y+100)/5 = 1/5 y + 20**

   The best way to describe this region is by thinking of it as a left-right region.

   On the left, we have x = 0

   On the right, we have x = 1/5 y + 20

Therefore, we have

$$\int_0^{25} \left(\int_0^{\frac{1}{5}y+20} -0.4\,x\,dx \right) dy = -741.\overline{6}\ \ \text{ft}^3$$